OPTIMIZING ZERO BETA PORTFOLIOS: A COMPARATIVE ANALYSIS OF ROBUST AND NORMAL PORTFOLIO METHODOLOGIES

OTIMIZAÇÃO DE PORTFÓLIOS ZERO BETA: UMA ANÁLISE COMPARATIVA DAS METODOLOGIAS DE PORTFÓLIO ROBUSTO E NORMAL

OPTIMIZACIÓN DE CARTERAS BETA CERO: UN ANÁLISIS COMPARATIVO DE LAS METODOLOGÍAS DE CARTERA ROBUSTA Y NORMAL

DOI: 10.56083/RCV4N3-107
Originals received: 02/01/2024
Acceptance for publication: 02/28/2024

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ABSTRACT: When building a “zero beta portfolio“, neglecting the parameters’ uncertainty may harm the investor. This paper analyzes a way to build a zero beta portfolio that does not consider only the parameter points estimates, but also the beta and the expected return uncertainties. The stocks’ betas and their uncertainties are calculated using the Kalman Filter and the stocks’ expected returns and their uncertainties are calculated from analysts’ price and dividends estimations. The study applied two different methodologies to build a zero beta portfolio: one that maximizes the ratio between the expected return by the uncertainties of the parameters, called long-short robust portfolio; and another that simply maximizes the expected return, neglecting the uncertainties of the parameters, called as long-short normal portfolio. During the period analyzed, 2015-2022, compared to the long-short normal portfolio, the long-short robust portfolio had a higher realized return and a significantly lower standard deviation.

RESUMO: Ao construir uma “carteira beta zero”, negligenciar a incerteza dos parâmetros pode ser prejudicial ao investidor. Este artigo analisa uma forma de construir uma carteira de beta zero que não considere apenas as estimativas pontuais dos parâmetros, mas também as incertezas do beta e do retorno esperado. No presente estudo, os betas das ações e suas incertezas são calculados usando o Filtro de Kalman e os retornos esperados das ações e suas incertezas são calculados a partir das estimativas de preços e dividendos dos analistas. O estudo aplicou duas metodologias distintas para a construção de uma carteira beta zero: uma que maximiza a razão entre o retorno esperado pelas incertezas dos parâmetros, denominada carteira robusta long-short; e outra que simplesmente maximiza o retorno esperado, desprezando as incertezas dos parâmetros, denominada carteira normal long-short. Durante o período analisado, 2015-2022, em comparação com a carteira normal long-short, a carteira robusta long-short apresentou maior retorno realizado e um desvio padrão significativamente menor.

PALAVRAS-CHAVE: Otimização Estocástica, Carteira Beta Zero, Finanças.
1. Introduction

In the realm of unrestricted arbitrage, an investor theoretically holds the potential of securing a risk-free profit, as illustrated in Figure 1. This could be attained when two well-diversified portfolios, both possessing identical betas, produce varying expected returns. The strategy involves shorting the portfolio that yields a lower expected return and procuring the one that offers a higher expected return.

However, it's essential to underscore that Figure 1 only presents the point estimate of the expected returns and beta of the portfolios, excluding their confidence intervals. Overlooking the uncertainty of these parameters may lead to severe drawbacks for the investor who shorts Portfolio A to purchase Portfolio B.

The expected return point estimate of Portfolio A presented in Picture 1 may be lower than Portfolio B. However, if both estimations have large
enough estimation errors it is reasonable that portfolio A could result in a higher realized return than portfolio B.

From the beta perspective, neglecting its estimation error may also be bad for an investor seeking a statistical arbitrage. If one (or both) portfolios shown in Picture 1 have beta with a large estimation error, what seems to be a statistical arbitrage opportunity may also result in a loss for the investor. To illustrate that, picture 2 shows an example that the real beta of portfolio A is lower than portfolio B – even though the point estimates were the same. In the example shown in picture 2, if an investor short Portfolio A and buy Portfolio B, in a case of a bear market, portfolio B may have a higher loss than Portfolio A, resulting in a loss to the investor.

Figure 2 - The risk of neglecting the estimations errors when pursuing an arbitrage

![Figure 2](source: self elaboration)

Therefore, from the expected return perspective, for a certain value of beta, when an investor aims to develop a statistical arbitrage position, it may be rational to build two portfolios with the farthest point estimation from another and with the lowest estimation error possible. On the other hand, from the beta perspective, it may be rational to pick two portfolios with the
closest point estimation possible and, also with low estimation error. Hence, when pursuing to analyze the feasibility of statistical arbitrage, it may be plausible to consider not only an accurate point estimation for the parameters but also their estimation errors. Then, this study aims to present a way to consider the parameters’ uncertainty when building portfolios with long and short positions.

It is worth mentioning that Göncü & Akyldirim (2016) and Anish (2021) state that once there is uncertainty about portfolio mean and standard deviation, statistical arbitrage is no longer a guaranteed approach, due to “error in trader’s guess or forecast of the long-term mean levels”. The expected profit is also based on the mean reversion price behavior (Ziping, Rui, & Palomar, 2019). About this problem, Do and Faff (2010) claim that there is a continuing downward trend in statistical arbitrage profitability. Beyond forecast errors that may cause a loss on long and short portfolios, Do & Faff (2010) state that arbitrageurs face other risks, such as “noise trader risk” – when illogical trading caused by noise traders prevent arbitrage approaches.

2. The Beta and Its Uncertainty Using the Kalman Filter

As a reminder, this study aims to build zero beta portfolios considering the uncertainties of the parameters. The Kalman Filter can be used as a tool to reduce the lack of precision caused by noise or other variables not considered in the valuation models, by minimizing the quadratic function of estimator error (Grewal and Andrews, 2014). By analyzing the characteristics of the problem presented in this paper, the Kalman Filter may be a useful tool to resolve that problem for beta estimation.
Neto (2014) states that an asset’s return is gauged by its value fluctuation plus the cash flow it generates. This research, therefore, proposes that a firm’s expected return is the expected change in its market capitalization, determined by the ratio of the analyst’s market cap forecast to the actual market cap at a given time \( t \), coupled with the projected dividends set by analysts. Furthermore, the standard deviation of all estimations - market cap and dividends - will quantify the uncertainties of these parameters.

Therefore, the expected excess return of an asset can be stated as presented in Equation 1.

\[
R_i = \frac{\hat{P} + \hat{D}}{P_0} + \beta_i \ast M
\]  

(1)

Where:

\( \hat{P} \) is the price target set by analysts, \( \hat{D} \) is the target dividend set by analysts.

The parameter uncertainties will be measured by the standard deviation of all estimates, where \( \hat{P} \) and \( \hat{D} \) will be set by the deviations of the analysts’ estimations and \( \beta_i \) will be the uncertainty calculated by the Kalman Filter. According to Xue, Di, & Zhang (2019), Qin, Kar, & Zheng (2016), Chen & Peng (2017), and Huang (2012) the security market is very complex and there are situations that historical data cannot be used to predict a security return and it is necessary to use expert’s estimation. Echterling, Eierle, & Ketterer (2015) affirm that a common method presented in financial literature to set the implied cost of capital is the usage of analyst forecasts. Bielstein & Hanauer (2019) states that one of the practical difficulties of
Markowitz’s mean-variance portfolio optimization is to estimate the stock’s expected return. For that parameter, the authors use analysts’ forecasts to estimate the stock’s expected return. Balakrishnan, Shivakumar, and Taori’s (2021) empirical study concludes that “analysts' cost of equity estimates are meaningful expected return proxies”. Fernandes, Ornelas, & Cusicanqui (2012) present a portfolio optimization technique that combines analysts’ expectations with estimations’ risk.

Zhai and Bai (2018) build a portfolio with experts’ opinions about the expected return, in which the returns distributions are considered as the securities’ expected return uncertainty. Xue, Di, and Zhang (2019) discuss portfolio selection under an environment of uncertainty in which the expected return is extracted by an expert’s estimation. Chen, Li, & Liu (2019), Chen & Peng (2017), and Huang (2012) portfolio selection articles consider experts’ estimations for the securities return and treat them as uncertain, with intervals instead of only a point estimate.

Fabozzi, Huang, & Zhou (2009) assert that the parameters estimate can be set by historical data or by expert prediction and, in the former case, instead of using the predictions of only one expert, it might be useful to combine the estimation from “several experts and consider each of their prediction as a likelihood distribution”. Goetzmann and Massa (2005) construct a portfolio considering the dispersion of stock return opinion. Rapach, Strauss, and Zhou recommend that the combination of numerous forecasts delivers better empirical out-of-sample equity premium predictions when compared to individual forecasts. Finally, Verardo (2009) measured the uncertainty about a firm fundamental by the dispersion in analyst forecasts.

Considering a naive approach where the uncertainties are independent, the sum of uncertain values can be calculated by adding the uncertainties. On the other hand, the product or quotient of uncertain values can be calculated by adding the fraction values uncertainties (Taylor, 1997).
Therefore, by applying those rules, the uncertainty of the expected excess return can be expressed as Equation 2.

\[
\Delta R_i = \frac{\Delta \rho}{\rho_o} + \frac{\Delta \beta_{t+1}}{\beta_{t+1}} + \frac{\Delta \mu}{\mu} + \frac{\Delta \mu}{M}
\]  

(2)

Where:

\(\Delta\) represents the parameters uncertainty. In addition, as \(\frac{\Delta \mu}{M}\) is the same for all assets, in this paper its value will be set as zero.

### 4. Building the “Zero Beta” Portfolio Considering the Uncertainties of the Parameters

The expected return of a portfolio is determined by the weighted mean of the predicted returns of each constituent asset. Correspondingly, the beta value of the portfolio for each factor is the weighted mean of the respective betas of each asset. Both the projected return and beta values pertain to point estimation. Yet, as alluded to in the study’s introduction, it may be prudent to acknowledge the inherent uncertainty of these parameters.

When building a portfolio with a long position and another for a short position aiming to conduct a statistical arbitrage process, it may also be prudent to consider values at the interval estimations, to be aware of the risks the investor will bear. Hence, in light of the mean-variance approach, originally introduced by Markowitz (1952), the optimal "zero beta" portfolio in this research will be identified as the one possessing the greatest ratio of expected return to its projected uncertainty. This approach underscores the significance of balancing potential returns and risk, paving the way for a more comprehensive and informed portfolio management strategy.
The “zero beta” portfolio means that the short and the long portfolios have the same beta value. In addition, the portfolio expected return will be set by the difference between the expected return of the long positions and the expected return of the short position.

Such as Markowitz (1952), the expected returns of the portfolios (either the long or the short ones) are set by the weighted average of the individual assets’ expected returns. Also, to pursue Markowitz’s (1952) maximum mean-variance, the variance of each asset is equal to the sum of the combined fractional uncertainty of its beta, market cap target, and dividends, as shown in the next section. Finally, this study will state a naive assumption for the covariance uncertainty, that the uncertainty of the parameters between the assets is independent.

5. The Data

Data elements such as market capitalization, target market capitalization, distributed and expected dividends, along with their corresponding estimation standard deviations, have been obtained from the Refinitive Eikon Database. In order to compute the betas via the Kalman Filter methodology, this research took into account the weekly percentage shift in the asset's value and the equally-weighted average return. We initiated the beta estimation by performing a linear regression from January 18, 2013, through December 27, 2013.

The coefficients obtained for each asset were subsequently implemented as an estimator for 2014. Additionally, we computed a fresh linear regression for each asset throughout 2014. Following that, the Kalman Filter, as described in section 2, was utilized in this research to amalgamate and estimate the beta for the successive years, in addition to their standard errors.
During portfolio construction, computational constraints to optimally weight the assets restricted the portfolio to only one hundred assets per annum. In this research, we first established the long and short portfolios at the onset of 2015 and subsequently updated them at the start of every subsequent year through 2022. Furthermore, upon the conclusion of each period, the portfolios will be assessed and juxtaposed to a portfolio that employs identical parameters but does not take into account their uncertainties.

6. The Results

As shown in Table 1, the actual realized return for each year differs substantially from the portfolio’s expected return. In all cases, the realized returns were much lower than the expected ones, and in some cases, the realized returns were even negative. Those results are consonant with the market efficiency hypothesis defined by Fama (1970), even for the weak form.

<table>
<thead>
<tr>
<th>Year</th>
<th>Long-Short robust portfolio expected return</th>
<th>Long-Short Robust Portfolio Realized Return</th>
<th>Long-Short normal portfolio expected return</th>
<th>Long-Short normal portfolio realized return</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>12.92%</td>
<td>0.91%</td>
<td>27.41%</td>
<td>1.96%</td>
</tr>
<tr>
<td>2016</td>
<td>12.41%</td>
<td>-2.90%</td>
<td>79.57%</td>
<td>-12.90%</td>
</tr>
<tr>
<td>2017</td>
<td>12.28%</td>
<td>4.49%</td>
<td>43.65%</td>
<td>1.16%</td>
</tr>
<tr>
<td>2018</td>
<td>10.69%</td>
<td>3.30%</td>
<td>30.60%</td>
<td>7.77%</td>
</tr>
<tr>
<td>2019</td>
<td>17.58%</td>
<td>-1.01%</td>
<td>70.57%</td>
<td>4.11%</td>
</tr>
<tr>
<td>2020</td>
<td>10.21%</td>
<td>-1.36%</td>
<td>45.54%</td>
<td>-4.78%</td>
</tr>
<tr>
<td>2021</td>
<td>9.78%</td>
<td>1.44%</td>
<td>43.78%</td>
<td>-2.29%</td>
</tr>
<tr>
<td>2022</td>
<td>11.55%</td>
<td>1.03%</td>
<td>44.62%</td>
<td>3.61%</td>
</tr>
</tbody>
</table>

Source: self elaboration

Columns 1 and 3 of Table 1 present the portfolio’s expected return after running the optimization tool. Additionally, columns 2 and 4 present the realized return of the respective portfolios. From now on, the “zero beta”
portfolios that maximized the ratio between the expected return by the parameters uncertainties will be called long-short robust portfolios, while the “zero beta” portfolios that simply maximized the expected return, neglecting the parameters uncertainties, will be called as long-short normal portfolio.

As expected, the long-short robust portfolio was more stable over time than the long-short normal portfolio. Picture 3 presents the box plot calculated from the weekly returns of both portfolios, where the blue box plot represents the weekly returns of the long-short robust portfolio and the orange box plot represents the weekly returns of the “long-short” normal portfolio.

Since the long-short robust portfolio had a higher accumulated realized return from 2015 to 2022 and a lower standard deviation, compared to the long-short normal portfolio, consequently the robust portfolio resulted in a higher ratio of realized return divided by the variance, as shown in table 2.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Total realized return</th>
<th>Standard deviation</th>
<th>Realized return divided by variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-Short Robust</td>
<td>6%</td>
<td>0.375%</td>
<td>4,145.86</td>
</tr>
<tr>
<td>Long-Short Normal</td>
<td>-3%</td>
<td>3.223%</td>
<td>(27.25)</td>
</tr>
</tbody>
</table>

Source: self elaboration
7. Conclusion and Future Studies

Even though the long-short robust portfolio developed in this study had a positive accumulated return from 2015-2022, during some years the returns were even negative, in consonant with the market efficiency hypothesis defined by Fama (1970). Considering the parameters uncertainties to build the portfolio seemed to be positive to reduce its standard deviation. Future studies may include another risk factor besides the market risk, such as the economic factor suggested by Chan, Roll, and Ross (1986) and/or the five-factor asset pricing model proposed by Fama, and French (2015). It may also be worthwhile to test in other markets as well as during long periods.
References


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